

## CLOSED LOOP SUPPLY CHAIN WITH PRODUCTION PLANNING

Wojciech Stecz\*

\* Faculty of Cybernetics Military University of Technology, Kaliskiego 2,  
00-908 Warsaw, Poland, Email: wstecz@wat.edu.pl

---

**Abstract** We present a Closed Loop Supply Chain (CLSC) model that supports a production planning (PP) process. CLSC model is based on CLSC framework model which consists of four main centers: collection, recovery center, distribution and disposal centers. These logistics parts support main production lines. Some quantity of the products is recovered and the factories don't need to spend money for production. This is a simple cost reduction process. In CLSC literature one can hardly meet the models of production planning processes supported by CLSC. Important problem with that models is the computational complexity when one wants to prepare production plans for more than one time period. This is connected with a number of the numerical variables of the CLSC and PP models which are usually Integer Programming models solved with Branch&Bound algorithms. We present some modifications of the widely known and used constraints in the CLSC models to optimize solving process. All the experiments were conducted with the CPLEX solver.

**Paper type: Research Paper**

**Published online:** 30 April 2016

Vol. 6, No. 2, pp. 117-128

**DOI:** 10.21008/j.2083-4950.2016.6.2.2

ISSN 2083-4942 (Print)

ISSN 2083-4950 (Online)

© 2016 Poznan University of Technology. All rights reserved.

**Keywords:** *Closed Loop Supply Chain, CLSC optimization, production planning*

---

## 1. INTRODUCTION

A supply chain forms a network of the collection, recovery and distribution centers that cooperate with the clients that use some products that can be recovered and used once again. Some clients return used products than should be recovered to be sold once again. These recovered products are cheaper than new products but are almost as good as new ones. So some companies, especially the printers producers, have been used Closed Loop Supply Chain to minimize their costs (Meade, Sarkis & Presley, 2007) (Amin & Zhang, 2013) (Cooper, Lambert & Pagh, 1997).

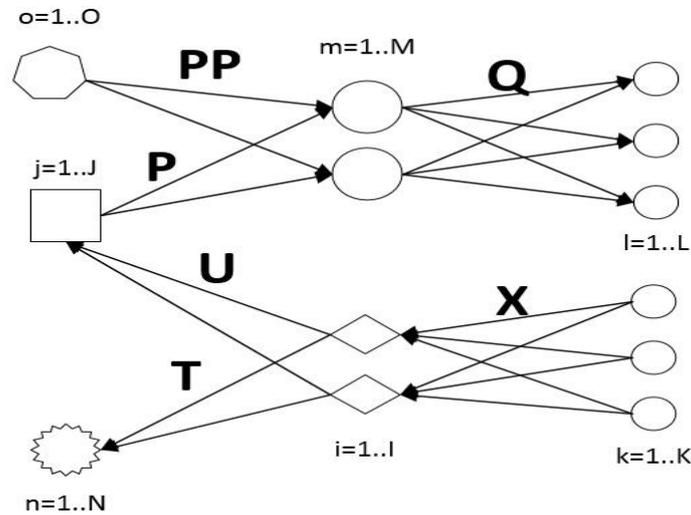
In recent decades companies as Xerox or Kodak have focused on remanufacturing and recovery activities to minimize their costs. There are two main factors that are driving forces of CLSC. First of all are business factors. The second factors are the environmental factors. The business factors concentrate mainly on production costs minimization. But the environmental factors could be also treated as funds consuming factors.

As the literature shows, most of the relevant works assume that all the parameters are deterministic. A comprehensive reviews on supply chain network design problems are presented in (Melo, Nickel & Saldanha-da-Gama, 2009) and (Klibi & Martel, 2010). A review of these problems is beyond the scope of the paper. Some problems with supply chain are presented in (Kramarz, 2015).

What is more important the process is calculated only for the one time period (for example one month). In our paper we use the deterministic parameters but our calculations are made for at least 5 time periods (we assumed that one period can describe a day or a week). This is a very important assumption because the models presented in the literature are the Integer Programming (IP) models. This means that even a simple IP model consists of many hundreds integer variables and some Branch&Bound methods should be used to find a solution of this model. When we extend our calculations to 5-30 time periods a number of IP variables is growing very fast. In our paper we show that even a medium size CLSC network generates many thousands of the IP variables and the B&B algorithms can hardly find the optimal solutions.

## 2. CLSC AND PRODUCTION PLANNING

Presented Closed Loop Supply Chain model is expanded by the addition of the production components. Base CLSC models describe only recovery processes without involvement of the production processes. But in this paper we treat a production process as a base business process. CLSC processes are the additional and supporting processes which allow the producer to minimize its costs when he uses recovered products or parts in his production. Fig. 1 describes our production and supply chain environment.



**Fig. 1** Closed loop supply chain supporting a production processes

Presented CLSC model is simplified and doesn't divide recovery processes into two parts: the products recovery and parts recovery. Sometimes some authors introduce a part recovery phase. This is a natural move when someone assumes that the parts of the products can be recovered and sold again. But in our model we omit this important assumption. Our model is based on (Pishvae, Rabbani & Torabi, 2011). We don't mention robust model capability but we introduce some modifications in the model formulation that improve a solving process. We correct also some errors in the reference model.

In our model a group of  $K$  clients return used products to a producer or its partner. These products are collected in the  $I$  collection centers, recovered at  $J$  recovery centers and moved to the  $M$  distribution centers. Then some  $L$  other clients can purchase them.

### 3. MODEL OF CLSC SUPPORTING A PRODUCTION PLANNING

Our Closed Loop Supply Chain is plausible for supporting a production planning process. Below we presented the mathematical formulation of this model. The model is based on the paper (Pishvae, Rabbani & Torabi, 2011), but we modified this by introduction of a production process. What is also important, we improved its effectiveness by addition of some constraints that improved the model quality. The model presented can be used to plan a production for more than one time unit. This is another additional feature of the presented model.

## Indices

$i$  – index of collection center  $i = 1..I$

$j$  – index of recovery center  $j = 1..J$

$m$  – index of redistribution center  $m = 1..M$

$n$  – index of disposal center  $n = 1..N$

$k$  – index of market customer zone – returns -  $k = 1..K$

$l$  – index of market customer zone (client demands)  $l = 1..L$

## Parameters

$d_{lt}$  – demand of customer  $l$  for the recovered products at time  $t$

$r_{kt}$  – return of the used products of a customer  $k$  at time  $t$

$cc_i$  – capacity of a collection center  $i$

$cr_j$  – capacity of a recovery center  $j$

$ce_m$  – capacity of a redistribution center  $m$

$cd_n$  – capacity of handling the scraped products at disposal center  $n$

$f_i$  – fixed cost of maintenance collection center  $i$

$g_j$  – fixed cost of maintenance recovery center  $j$

$h_m$  – fixed cost of maintenance redistribution center  $m$

$a_{ijt}$  – shipping cost per unit of recoverable products from collection center  $i$  to recovery center  $j$  at time  $t$

$b_{jmt}$  – shipping cost per unit of recovered products from recovery center  $j$  to redistribution center  $m$  at time  $t$

$e_{mlt}$  – shipping cost per unit of recovered products from redistribution center  $j$  to customer zone  $l$  at time  $t$

$v_{int}$  – shipping cost per unit of scrapped products from collection center  $i$  to disposal center  $n$  at time  $t$

$c_{kit}$  – shipping cost per unit of returned products from customer zone  $k$  to collection center  $i$  at time  $t$

$\pi_l$  – penalty cost per unit of non-satisfied demand of customer  $l$

$strc_t$  – storage costs at time  $t$  – identical for the CLSC elements

$strp_t$  – storage costs at time  $t$  for the production warehouses

## Variables

$X_{kit}$  – quantity of the returned products shipped from the customer zone  $k$  to the collection center  $i$  at time  $t$

$U_{ijt}$  – quantity of the recoverable products shipped from the collection center  $i$  to the recovery center  $j$  at time  $t$

$P_{jmt}$  – quantity of the recovered products shipped from the recovery center  $j$  to the redistribution center  $m$  at time  $t$

$Q_{mlt}$  – quantity of the recovered products shipped from the redistribution center  $m$  to the customer zone  $l$  at time  $t$

$T_{int}$  – quantity of the scrapped products shipped from the collection center  $i$  to the disposal center  $n$  at time  $t$

- $\delta_{lt}$  – quantity of non-satisfied demand of customer  $l$  at time  $t$   
 $Y_{it}$  – 1 if a collection center is opened at location  $i$  at time  $t$   
 $Z_{jt}$  – 1 if a recovery center is opened at location  $j$  at time  $t$   
 $W_{mt}$  – 1 if a redistribution center is opened at location  $m$  at time  $t$   
 $sco_{it}$  – storage level in the collection center  $i$  at time  $t$   
 $src_{jt}$  – storage level in the recovery center  $j$  at time  $t$   
 $scd_{mt}$  – storage level in the distribution  $m$  center at time  $t$   
 $XP_{ot}$  – production at time  $t$  in the production center  $o$   
 $YP_{ot}$  – 1 if the production center  $o$  is open at time  $t$   
 $PP_{ot}$  – demand for the produced items at time  $t$   
 $sp_{ot}$  – storage level in the collection center  $o$  at time  $t$

### Model formulation

The components of the minimization function:

$$SumInspectionCenter = \sum_{i \in I, t \in Pd} f_i * Y_{it}$$

Fixed cost of a collection center maintenance per time unit.

$$SumRecoveryCenter = \sum_{j \in J, t \in Pd} g_j * Z_{jt}$$

Fixed cost of a recovery center maintenance per time unit.

$$SumRedistributionCenter = \sum_{m \in M, t \in Pd} h_m * W_{mt}$$

Fixed cost of a redistribution center maintenance per time unit.

Shipping costs of the products among the CLSC components (see Fig. 1):

$$Sum\_FC\_IC = \sum_{k \in K, i \in I, t \in Pd} c_{kit} * X_{kit}$$

$$Sum\_IC\_RC = \sum_{i \in I, j \in J, t \in Pd} a_{ijt} * U_{ijt}$$

$$Sum\_RC\_RDC = \sum_{j \in J, m \in M, t \in Pd} b_{jmt} * P_{jmt}$$

$$Sum\_RDC\_SC = \sum_{m \in M, l \in L, t \in Pd} e_{mlt} * Q_{mlt}$$

$$Sum\_IC\_DC = \sum_{i \in I, n \in N, t \in Pd} v_{int} * T_{int}$$

Penalty costs when a demand is greater than a sum of a production and a recovery.

$$Sum\_L\_Penalty = \sum_{i \in L, t \in Pd} \pi_i * \delta_{it}$$

Production costs – set up for the experiments in the paper.

$$Sum\_L\_Prod = \sum_{o \in O, t \in Pd} g_1 * (XP_{ot} + YP_{ot})$$

Storage costs (only production storages can have different costs during the planning period).

$$Sum\_Storage = \sum_{i \in I, t \in Pd} storage_t * sco_{it}) + \sum_{j \in J, t \in Pd} storage_t * src_{jt}) \\ + \sum_{m \in M, t \in Pd} storage_t * scd_{mt}) + \sum_{o \in O, t \in Pd} storage_{p_{ot}} * sp_{ot}) +$$

Optimization goal:

minimize

SumInspectionCenter + SumRecoveryCenter + SumRedistributionCenter +  
Sum\_FC\_IC + Sum\_IC\_RC + Sum\_RC\_RDC + Sum\_RDC\_SC + Sum\_IC\_DC +  
Sum\_L\_Penalty + Sum\_storage + Sum\_L\_Prod

Constraints:

$$\sum_{m \in M} Q_{mIt} = d_{It} - \delta_{It}, \forall I \in L, t \in Pd$$

A number of the products recovered and produced is equal to the difference between demand and non-satisfied demand (for any time unit).

$$\sum_{i \in I} X_{kit} = r_{kt}, \forall k \in K, t \in Pd$$

A quantity of the returned products is equal to the quantity of the products sent to the collection centers.

$$\sum_{j \in I} U_{ijt} + \sum_{n \in N} T_{int} + sco_{it} = \sum_{k \in K} X_{kit} + sco_{i,t-1}, \forall i \in I, t \in Pd$$

State equality. A quantity of the products collected in the collection centers is equal to the quantity of the products sent later to the recovery and the disposal centers (taking into account the storage levels of the products in the collection centers warehouses in  $t-1$  and  $t$ ).

$$\sum_{n \in N} T_{int} \geq s * \sum_{k \in K} X_{kit}, \forall i \in I, t \in Pd$$

A quantity of the products sent to the disposal centers from the collection centers is greater or equal as  $s*100\%$ .  $s$  is set up as a-priori known parameter.

$$\sum_{n \in N} T_{int} \leq \sum_{k \in K} X_{kit}, \forall i \in I, t \in Pd$$

A quantity of the products sent to the disposal centers from the collection centers is less or equal than a quantity of the products collected.

$$\sum_{j \in I} P_{jmt} + scd_{m,t-1} + \sum_{o \in O} PP_{omt} = \sum_{l \in L} Q_{mlt} + scd_{mt}, \forall m \in M, t \in Pd$$

State equality. A quantity of the recovered products shipped from the recovery centers to the redistribution centers plus a quantity of produced goods shipped to the redistribution centers are equal to a quantity of the products sent to the clients (taking into account the storage levels at  $t-1$  and  $t$ ).

$$sp_{o,t-1} + XP_{ot} = \sum_{m \in M} PP_{omt} + sp_{ot}, \forall o \in O, t \in Pd$$

State equality for the production centers. Production should be equal to the demand, taking into account the storage levels of the production warehouses.

$$XP_{ot} \leq \sum_{l \in L, tt \in Pd: tt \geq t} d_{l,tt} * YP_{ot}, \forall o \in O, t \in Pd$$

At time  $t$  a quantity of the produced goods is less or equal to a quantity of the demands for the remaining time units.

$$XP_{ot} \leq \sum_{l \in L, tt \in Pd: tt \geq t} d_{l,tt}, \forall o \in O, t \in Pd$$

Technical inequality improving a solver searching process. Almost the same as above, but 0-1 variable was omitted.

$$\sum_{o \in O} XP_{ot} \leq \sum_{l \in L, tt \in Pd: tt \geq t} d_{l,tt}, \forall t \in Pd$$

Technical inequality improving a solver searching process. Almost the same as above, but the whole production quantity is compared to the demands.

$$\sum_{o \in O, m \in M} PP_{omt} + \sum_{j \in J, m \in M} P_{jmt} \leq \sum_{l \in L, tt \in Pd: tt \geq t} d_{l,tt}, \forall t \in Pd$$

A sum of a quantity of the produced goods and the recovered goods at  $t$  is less or equal to the demands for the remaining time periods.

$$\sum_{o \in O, m \in M} PP_{omt} + \sum_{j \in J, m \in M} P_{jmt} \geq \sum_{l \in L} d_{l,t}, \forall t \in Pd$$

A sum of a quantity of the produced goods and the recovered goods at  $t$  is greater or equal to the demands at  $t$ .

$$\sum_{m \in M} P_{jmt} + src_{jt} = \sum_{i \in I} U_{ijt} + src_{j,t-1}, \forall j \in J, t \in Pd$$

State equality. A quantity of products recovered and sent to the distribution centers is equal to a quantity of the products sent to the recovery centers taking into account the storage levels a  $t-1$  and  $t$ .

Below are the equalities that constrain the center's capacities. These capacities are modeled as the spaces needed for collection, remanufacturing and distribution activities and are different than the storage capacities of these centers.

$$\sum_{k \in K} X_{kit} \leq Y_{it} * cc_i, \forall i \in I, t \in Pd$$

$$\sum_{i \in I} U_{ijt} \leq Z_{jt} * cr_j, \forall j \in J, t \in Pd$$

$$\sum_{j \in I} P_{jmt} \leq W_{mt} * ce_m, \forall m \in M, t \in Pd$$

$$\sum_{i \in I} T_{int} \leq cd_n, \forall n \in N, t \in Pd$$

The last inequality could be omitted if we assume that the broken goods don't need the special warehouses to be collected before disposal.

#### 4. OPTIMIZATION RESULTS

We calculated the production planning problem with a CLSC support for the different demands and the different time periods. But the most important factor influencing the results was the CLSC construction, i.e. the numbers of the collection centers, the recovery centers and the distribution centers. Some of the numerical experiments are shown below. CPLEX solver used Branch&Bound algorithm supported but cuts and pricing (Nocedal & Wright, 1999) (Nemhauser & Wolsey, 1988) (Wolsey & Neumhauser, 1999).

Experiment 1

Parameters:

Collection center	I = 18
Recovery center	J = 12
Redistribution center	M = 18
Disposal centers	N = 2
First market customer zones	K = 30
Second market customer zones (demands)	L = 50
Optimization period	T = 6

ILOG CPLEX results:

Variables = 12576

Cuts generated during optimization:

Implied bound cuts applied:	654
Flow cuts applied:	311
Mixed integer rounding cuts applied:	762
Flow path cuts applied:	101
Zero-half cuts applied:	1
Lift and project cuts applied:	42
Gomory fractional cuts applied:	37
Total (root+branch&cut) =	337.32 sec.

CPLEX solver found the result quickly because of a small number of logistics centers. One should spot that during optimization when solver was constructing the branch and bound tree, many cuts were generated. When the number of the logis-

tics centers was increased, the CPLEX solver wasn't able to find the solution in one hour (using the predefined CPLEX solver result gap – 0.01). But when one doesn't need the optimal solution then the result gap parameter can be changed to 0.02 and results are found quickly.

#### Experiment 2

The same input parameters were used but we changed solver configurations. We modified parameters:

CPX\_PARAM\_MIPORDTYPES

CPX\_PARAM\_MIPEMPXASIS

CPX\_PARAM\_MIPORDTYPES

and compared these results with the standard configuration. Results presented below show that the standard configuration is the best for the most of CLSC models.

**Table 1**

No of weeks	No of variables	Time in [s]		
		Predefined configuration	CPX_PARAM_MIPORDTYPES: cost minimization	CPX_PARAM_MIPORDTYPES: cost minimization CPX_PARAM_MIPEMPXASIS: Feasibility over optimality
1	2096	12	16	2
2	4192	17	10	47
3	6288	23	19	42
4	8384	54	36	83
5	10480	580	>2400	

#### Experiment 3

When additional technical constraints were added to the model, the results were found faster. But when one had to calculate production planning model for 7-8 weeks, the results were presented after one hour. In a real environment when there are many of the logistics centers one should calculate his numerical experiments for up to 5 weeks. This results were obtained with a two core processor Intel i3. But in a real multiprocessor environments these results were presented faster.

## 5. CONCLUSION

In our paper we presented a modified CLSC network that consists of the CLSC elements and the production elements. We assumed that CLSC recovery process is a supporting process and the production process is a main process analyzed in

the paper. Our calculations were extended to more than one time period what had an impact onto a solution finding process. We used a CLEX solver. The presented model was IP model.

We presented some modifications of the base CLSC model to improve a calculation speed. Some computational results were presented. These results show that one should take into account a possibility of relaxation of the variables describing a number of the recovered and the collected products. This assumption can be easily explained when we notice that many collected products can't be recovered later. So this assumption is true in a practice. And improve solution finding process.

## REFERENCES

- Amin, S., H. & Zhang, G., (2013), A multi-objective facility location model for closed-loop supply chain network under uncertain demand and return, *Applied Mathematical Modelling*, Vol. 37, pp. 4165–4176.
- Cooper M.C., Lambert D. & Pagh, J.D., (1997), Supply chain management: more than a new name for logistics, *International Journal of Logistics Management*, Vol. 8, No. 1, pp. 1-9.
- Klibi W., Martel A. & Guitouni, A., (2010), The design of robust value-creating supply chain networks: a critical review, *Eur. J. Oper. Res.*, Vol. 203, pp. 283–293.
- Kramarz W., Kramarz M., (2015), Strategy of improving resistance of supply chain in conditions of disruptions, *Research in Logistics & Production*, Vol. 1, pp. 53-64.
- Meade L., Sarkis J. & Presley A., (2007), The theory and practice of Reverse Logistics, *Int. J. Log. Syst. Manage.* Vol. 3, pp. 56–84.
- Melo M.T., Nickel S. & Saldanha-da-Gama F., (2009), Facility location and supply chain management – a review, *Eur. J. Oper. Res.*, Vol. 196, pp. 401–412.
- Nemhauser G.L. & Wolsey L.A., (1988), "Integer and Combinatorial Optimization", John Wiley & Sons, New York.
- Nocedal J. & Wright S.J., (1999), "Numerical optimization", Springer–Verlag, New York.
- Pishvae M.S., Rabbani M. & Torabi S.A., (2011), A robust optimization approach to closed-loop supply chain network design under uncertainty, *Applied Mathematical Modelling* Vol. 35, pp. 637–649.
- Uster H., Easwaran G., Akcali E. & Cetinkaya S., (2007), Benders decomposition with alternative multiple cuts for a multi-product closed-loop supply chain, network design model, *Naval Research Logistics*, Vol. 54, pp. 890–907.
- Wolsey L.A. & Neumhauser N.L., (1999), "Integer and combinatorial optimization", Wiley-Interscience.

## BIOGRAPHICAL NOTES

**Wojciech Stecz** is an assistant professor at the Faculty of Cybernetics at Military University of Technology in Warsaw. He obtained his PhD at MUT in 2004 in the subject of information systems. Between 1997 and 1999 he was working at Institute of Command Systems Automation at MUT. From 2000 up to now he has been working as an assistant professor at Faculty of Cybernetics.

He is an author of several articles in the scope of optimization of the business processes in industry and logistics. He has been working as an ERP and the business processes consultant in industry and finance.