MINIMIZATION OF EMPTY TRUCK RUNS. FORMULATION OF PROBLEM

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Abstract The article formulates the issue of minimizing empty runs. Then, the interpretation of the issue of empty runs on numerical examples is presented. The formulation of the issue of minimizing empty runs requires the formulation of two optimization problems. The first one concerns distribution of commodities with minimal cost. The optimization problem formulated in this way is a well-known transport problem with the objective function being the costs of transport. The second one concerns commodity transport with distance minimization. After solving these two issues, the circulation problem of transport means is solved. Thanks to which we obtain the minimum number of empty runs expressed in distance units.

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1. INTRODUCTION

Proper route planning and minimization of empty runs can significantly reduce costs and above all, increase the efficiency of the transport company. Empty runs, in the sense of unloaded means of transport, increase the cost of services and negatively affect the environment. This is an adverse phenomenon but unfortunately inevitable. According to a report from 2017, published by the European Commission, approximately 23% of all truck runs in the EU were empty runs (European Commission, 2017). Therefore, it is important to limit them to a minimum by optimizing routes planning, taking into account the minimization of empty runs with the full use of transport possibilities. The article formulates the optimization problem of determining the minimum circuits of transport means. The input data necessary to formulate the optimization problem is assumed. The decision variables and the analytical form of constraints at which routes are resolved in the sense of the given criterion function are determined. The model is based on Ambroziak & Żak (2004).

2. PROBLEM OF EMPTY TRUCK RUNS

2.1. Defined values

The demand for transport services will be expressed by a matrix with elements z_{ij}. Which have interpretation of the demand for transport between the distinguished suppliers D_i and recipients O_j where $i = 1, 2, \ldots, I$ and $j = 1, 2, \ldots, J$.

Let the matrix Z with elements z_{ij} has the form (1):

$$Z = \begin{bmatrix} z_{11} & \dots & z_{1j} & \dots & z_{1J} \\ \dots & \dots & \dots & \dots & \dots \\ z_{i1} & \dots & z_{ij} & \dots & z_{iJ} \\ \dots & \dots & \dots & \dots & \dots \\ z_{I1} & \dots & z_{Ij} & \dots & z_{IJ} \end{bmatrix} \tag{1}$$

Transport is considered within the area determined by cargo providers D_i and cargo recipients O_j. There is also a distance matrix d_{ij} between individual suppliers and recipients as below (2):

$$D = \begin{bmatrix} d_{11} & \dots & d_{1j} & \dots & d_{1J} \\ \dots & \dots & \dots & \dots & \dots \\ d_{i1} & \dots & d_{ij} & \dots & d_{iJ} \\ \dots & \dots & \dots & \dots & \dots \\ d_{I1} & \dots & d_{Ij} & \dots & d_{IJ} \end{bmatrix} \tag{2}$$

It is obvious that adequate number of transport units of a certain type (all with the load capacity equal to q) should be used to meet transport needs.

Based on the matrix Z a matrix rz_{ij} for work courses can be easily created. Elements of the matrix Z can be expressed by the number of transport units (each with a capacity of q) necessary to transport the cargo between the suppliers D_i and the recipients O_j. In this case the matrix RZ with elements rz_{ij} has the form (3):

$$RZ = \begin{bmatrix} rz_{11} & \dots & rz_{1j} & \dots & rz_{1J} \\ \dots & \dots & \dots & \dots & \dots \\ rz_{i1} & \dots & rz_{ij} & \dots & rz_{iJ} \\ \dots & \dots & \dots & \dots & \dots \\ rz_{I1} & \dots & rz_{iJ} & \dots & rz_{IJ} \end{bmatrix} \tag{3}$$

The elements of the above matrix can be expressed by the formula (4):

$$rz_{ij} = \{\tau : \tau \cdot q - z_{ij} \geq 0\} \quad where\ \tau = 1, 2, \dots \tag{4}$$

where element rz_{ij} is a number which has an interpretation of full cargo transport from the supplier D_i to the recipient O_j.

2.2. Searched values (decision variables of the empty truck runs)

Let the element xz_{ji} has an interpretation of the number of transport units that should be in the recipient O_j for providing transport capabilities from the supplier D_i.

From the supplier D_i has to leave (5):

$$\tau_i = \sum_{j=1}^{J} xz_{ji} \quad where\ i = 1, 2, \dots, I \tag{5}$$

loaded transport units.

In the same period of time to supplier D_i has to arrive the same number of empty means of transport. The following equation is met (6):

$$\sum_{j=1}^{J} xz_{ji} = \tau_i = \sum_{j=1}^{J} rz_{ij} \quad where\ i = 1, 2, \dots, I \tag{6}$$

Similarly in the same period of time to the recipient O_j should arrive certain amount of (7):

$$\tau_j = \sum_{i=1}^{I} rz_{ij} \quad where\ j = 1, 2, \dots, J \tag{7}$$

loaded transport units, whereas the same amount of empty ones can therefore depart from the recipient O_j. The second condition limiting the problem of empty runs is the following statement (8):

$$\sum_{i=1}^{I} xz_{ji} = \tau_j = \sum_{i=1}^{I} rz_{ij} \quad where\ j = 1, 2, \dots, J \tag{8}$$

The total number of kilometres completed by empty transport units will be (9):

$$\sum_{i=1}^{I}\sum_{j=1}^{J} d_{ji} \cdot xz_{ji} \tag{9}$$

From the above it follows that the problem of empty runs can be formulated as follows. The distance function should be minimized in the form (10):

$$F(X) = \sum_{i=1}^{I}\sum_{j=1}^{J} d_{ji} \cdot xz_{ji} \tag{10}$$

subject to:

$$\sum_{j=1}^{J} xz_{ji} = \sum_{j=1}^{J} rz_{ij} \quad where\ i = 1, 2, \dots, I \tag{11}$$

$$\sum_{i=1}^{I} xz_{ji} = \sum_{i=1}^{I} rz_{ij} \quad where\ j = 1, 2, \dots, J \tag{12}$$

As it can be seen, the above problem is a typical linear programming problem, known in the literature as the transportation problem with the distance criterion. The solution to this problem (e.g. The Stepping-Stone Solution Method) is a matrix $XZ = [X^{*}_{ij}]$ of optimal empty truck runs.

2.3. Numerical interpretation of the issue of empty runs on numerical example

Matrix Z is given and transport unit with a capacity of $q = 2T$.

$$Z = \begin{bmatrix} 0 & 0 & 0 & 13 & 0 \\ 19 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 & 15 \\ 0 & 15 & 8 & 2 & 0 \end{bmatrix} \tag{13}$$

From the above it follows that $rz_{14} = 13$ tonnes of cargo should be transported from the supplier D_1 to the recipient O_4, similarly $rz_{52} = 15$ tonnes of cargo should be transported from the supplier D_5 to the recipient O_2.

There is a distance matrix D between the suppliers D_i and the recipients O_j (14).

$$D = \begin{bmatrix} 459 & 170 & 634 & 621 & 615 \\ 602 & 406 & 78 & 93 & 53 \\ 460 & 224 & 251 & 236 & 232 \\ 507 & 210 & 433 & 440 & 413 \\ 514 & 436 & 59 & 22 & 52 \end{bmatrix} \tag{14}$$

It follows that the distance between supplier D_1 and the recipient O_5 is $d_{15} = 514$ kilometres. Analogously in other cases.

A matrix $[rz_{ij}]$ containing a demand for full cargo transport is created using the formula (15):

$$rz_{ij} = \{\tau : \tau \cdot q - z_{ij} \geq 0\} \quad where\, \tau = 1, 2, \ldots \tag{15}$$

$$RZ = \begin{bmatrix} 0 & 0 & 0 & 7 & 0 \\ 10 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 0 & 8 \\ 0 & 8 & 4 & 1 & 0 \end{bmatrix}$$

The demand for work courses can be presented in a graphical form as in Fig. 1.

It follows that the number of working courses (with full load) from the supplier D_1 to the recipient O_4 is $rz_{14} = 7$ transport units, six full courses (with $q = 2T$) and one incomplete. Analogous interpretation for the remaining elements of the RZ matrix.

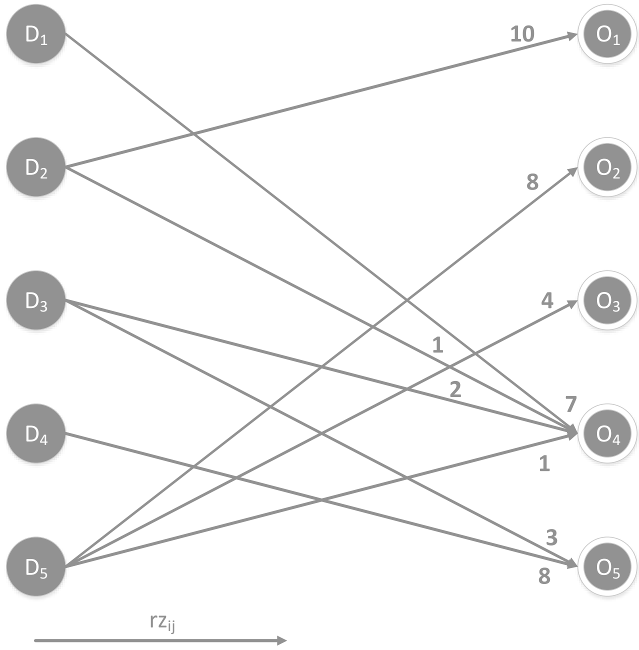

Fig. 1 A diagram of demands rz_{ij} for full transport unit courses

A plan of empty means of transport necessary to carry out the planned transport should be prepared, assuming that the number of empty runs of transport means will be minimal. Such an issue can be presented using the classical transportation problem.

Table 1 The tableau to solve the transportation problem with the distance criterion

	O_1	O_2	O_3	O_4	O_5	
D_1	459	170	634	621	615	7
D_2	602	406	78	93	53	11
D_3	460	224	251	236	232	5
D_4	507	210	433	440	413	8
D_5	514	436	59	22	52	13
	10	8	4	11	11	

The last row shows the demand of individual recipients for empty runs. In the last column the number of empty runs is given for subsequent suppliers. The remaining elements of the table contain distances between individual suppliers and recipients.

The objective function showing the number of empty runs of transport means that should be minimized takes the form:

$$F(X) = \sum_{i=1}^{5}\sum_{j=1}^{5} d_{ij} \cdot xz_{ji} = 459 \cdot xz_{11} + 170 \cdot xz_{12} + 634 \cdot xz_{13} + 621 \cdot xz_{14} + 615 \cdot xz_{15} + 602 \cdot xz_{21} + 406 \cdot xz_{22} + 78 \cdot xz_{23} + 93 \cdot xz_{24} + 53 \cdot xz_{25} + 460 \cdot xz_{31} + 224 \cdot xz_{32} + 251 \cdot xz_{33} + 236 \cdot xz_{34} + 232 \cdot xz_{35} + 507 \cdot xz_{41} + 210 \cdot xz_{42} + 433 \cdot xz_{43} + 440 \cdot xz_{44} + 413 \cdot xz_{45} + 514 \cdot xz_{51} + 436 \cdot xz_{52} + 59 \cdot xz_{53} + 22 \cdot xz_{54} + 52 \cdot xz_{55}$$

subject to:

$$\sum_{j=1}^{5} xz_{ji} = \sum_{j=1}^{5} rz_{ij} \quad where\ i = 1,2,3,4,5$$

$xz_{11} + xz_{21} + xz_{31} + xz_{41} + xz_{51} = rz_{11} + rz_{12} + rz_{13} + rz_{14} + rz_{15} = 7$ where i = 1
$xz_{12} + xz_{22} + xz_{32} + xz_{42} + xz_{52} = rz_{21} + rz_{22} + rz_{23} + rz_{24} + rz_{25} = 11$ where i = 2
$xz_{13} + xz_{23} + xz_{33} + xz_{43} + xz_{53} = rz_{31} + rz_{32} + rz_{33} + rz_{34} + rz_{35} = 5$ where i = 3
$xz_{14} + xz_{24} + xz_{34} + xz_{44} + xz_{54} = rz_{41} + rz_{42} + rz_{43} + rz_{44} + rz_{45} = 8$ where i = 4
$xz_{15} + xz_{25} + xz_{35} + xz_{45} + xz_{55} = rz_{21} + rz_{22} + rz_{23} + rz_{24} + rz_{25} = 13$ where i = 5

$$\sum_{i=1}^{5} xz_{ji} = \sum_{i=1}^{5} rz_{ij} \quad where\ j = 1,2,3,4,5$$

$xz_{11} + xz_{12} + xz_{13} + xz_{14} + xz_{15} = rz_{11} + rz_{21} + rz_{31} + rz_{41} + rz_{51} = 10$ where j = 1
$xz_{21} + xz_{22} + xz_{23} + xz_{24} + xz_{25} = rz_{12} + rz_{22} + rz_{32} + rz_{42} + rz_{52} = 8$ where j = 2
$xz_{31} + xz_{32} + xz_{33} + xz_{34} + xz_{35} = rz_{13} + rz_{23} + rz_{33} + rz_{43} + rz_{53} = 4$ where j = 3
$xz_{41} + xz_{42} + xz_{43} + xz_{44} + xz_{45} = rz_{14} + rz_{24} + rz_{34} + rz_{44} + rz_{54} = 11$ where j = 4
$xz_{51} + xz_{52} + xz_{53} + xz_{54} + xz_{55} = rz_{15} + rz_{25} + rz_{35} + rz_{45} + rz_{55} = 11$ where i = 5

The optimal solution in the form of a matrix $[\dot{X_{ij}}]$ is obtained when the presented transportation problem is solved.

$$[\dot{X_{ij}}] = \begin{bmatrix} 7 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 2 & 0 & 2 \\ 0 & 8 & 0 & 0 & 3 \\ 0 & 0 & 3 & 8 & 0 \end{bmatrix}$$

The number of empty runs is determined using the objective function:

$$F(X) = \sum_{i=1}^{5}\sum_{j=1}^{5} d_{ij} \cdot xz_{ji}$$
$$= 459 \cdot 7 + 170 \cdot 3 + 634 \cdot 0 + 621 \cdot 0 + 615 \cdot 0 + 602 \cdot 0 + 406 \cdot 0 + 78 \cdot 0 + 93 \cdot 0 + 53 \cdot 8 + 460 \cdot 0 + 224 \cdot 0 + 251 \cdot 2 + 236 \cdot 0 + 232 \cdot 2 + 507 \cdot 0 + 210 \cdot 8 + 433 \cdot 0 + 440 \cdot 0 + 413 \cdot 3 + 514 \cdot 0 + 436 \cdot 0 + 59 \cdot 3 + 22 \cdot 8 + 52 \cdot 0 = 8385$$

The optimal solution can be presented in graphic form as in Fig. 2.

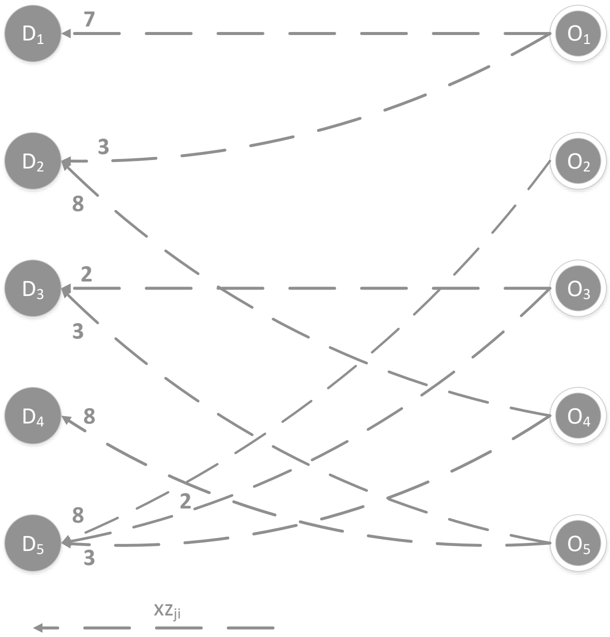

Fig. 2 A diagram of demands xz_{ij} for empty transport unit courses

The matrices *RZ* and *X* are subjected to the merge process. The result of which is the graph of demand for full and empty transport unit courses shown in Fig. 3.

The resulting graph Fig. 3 makes it possible to start generating routes. Each route is a set of elements arranged in the order in which particular supplier/recipient will be visited by transport units moving along a given route. The route has a following property – the beginning of the route coincides with its end. Such routes in graph theory are called cycles.

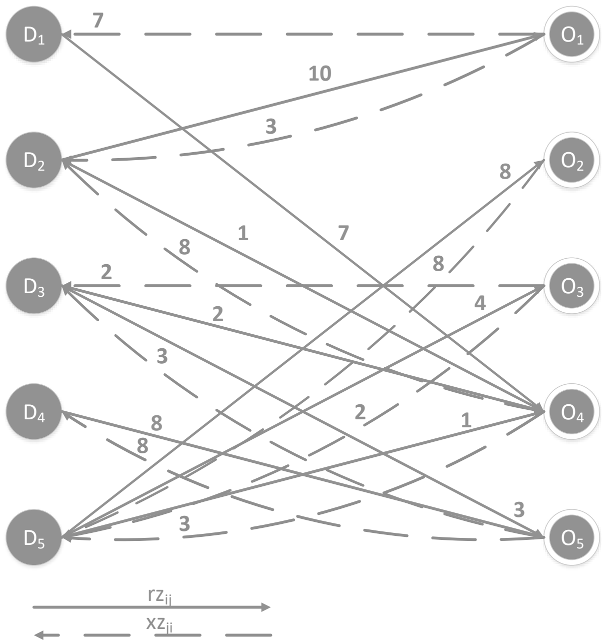

Fig. 3 The demand for full rz_{ij} and empty xz_{ij} transport unit courses

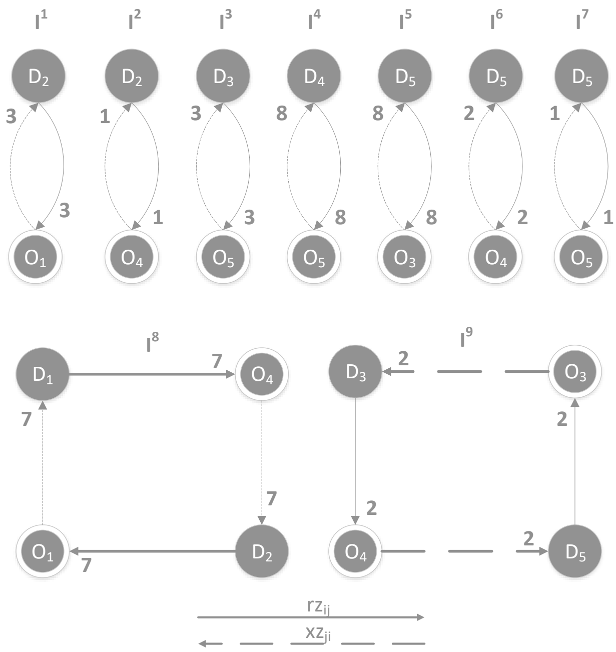

Fig. 4 The optimal routes plan

The optimal routes plan Fig. 4 which minimizes empty runs was determined using the generated routes.

Taking into consideration e.g. route I^8 from Fig. 4 it shows that from supplier D_1 7 full cargo transport will be departed to the recipient O_4. Then from the recipient O_4 to the supplier D_2 will be departed 7 empty means of transport. From the supplier D_2 7 full means of transport will be departed to the recipient O_1. The last part of the route i.e. from the recipient O_1 to the supplier D_1 will be departed 7 empty means of transport. The other routes should be interpreted in the same way.

3. CONCLUSION

The results presented in this work were generated using proprietary computer software. The time needed to determine the optimal solution for large set of input data, using the algorithm discussed in the article, can be quite long. It follows that it is necessary to work on improving the algorithms, and more specifically on the speed of their operation in solving complicated problems. The next step would be to develop a specialized software package that would allow automatic visualization of the generated routes, based on Lipski (1989), Piasecki (1973), Schrijver (2003), Trzaskalik (2008).

The mathematical model presented in the article unambiguously confirms the validity of the optimization of empty runs. In the presented optimization problem, empty courses were reduced from 8443 to 8385 kilometres therefore the proposed approach is beneficial for reducing costs including transport, operation or maintenance.

REFERENCES

Ambroziak T. & Żak J. (2004) Metoda wyznaczenia optymalnej liczby środków transportowych dla realizacji określonego zadania transportowego, Wydawnictwo Politechniki Śląskiej, Gliwice.

European Commission – DG for Mobility and Transport (2017) An Overview of the EU Road Transport Market in 2015, available at: https://ec.europa.eu/transport/sites/transport/files/mobility-package-overview-of-the-eu-road-transport-market-in-2015.pdf (accessed 17 March 2019).

Kryński H. & Badach A. (1976) Zastosowania matematyki do podejmowania decyzji ekonomicznych, Państwowe wydawnictwo naukowe, Warszawa.

Lipski W. (1989) Kombinatoryka dla programistów, Wydawnictwa Naukowo-Techniczne, Warszawa.

Piasecki S. (1973) Optymalizacja systemów przewozowych, Wydawnictwa Komunikacji i Łączności, Warszawa.

Schrijver A. (2003) Combinatorial Optimization. Polyhedra and Efficiency, Springer-Verlag, Berlin Heidelberg.

Trzaskalik T. (2008) Wprowadzenie do badań operacyjnych z komputerem, Polskie Wydawnictwo Ekonomiczne, Warszawa.

BIOGRAPHICAL NOTES

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