RESEARCH IN LOGISTICS & PRODUCTION

FORMULATION OF OPTIMISATION TASK FOR CARGO TRANSPORT IN ESTABLISHED REGROUP RELATIONS

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Abstract The following paper is connected to the formulation of the task for traffic flow optimisation in the examined part of transport network for a criterion of equal average costs. It is assumed that transport network is presented in a form of a graph. The arcs of a graph have interpretation of paths connections, vertices of a graph have interpretation of locations. On paths connections, the capacity function interpretation has been defined. Based on information mentioned above, the optimization task of determining the equipment regrouping relations is formulated. First of all, assumptions on displacement of vehicles in a given network in a regroup relation were defined. What is more, criteria of vehicles flows in regroup relations in a transport network were formulated and then optimizing tasks for vehicles flow were defined: the first optimization task of vehicles flow with average cost criterion and the second one: optimization task of vehicles flow with cost criterion.

Paper type: Research Paper

Published online: 15 April 2019
Vol. 9, No. 2, pp. 113–123
DOI: 10.21008/j.2083-4950.2019.9.2.5

ISSN 2083-4942 (Print)
ISSN 2083-4950 (Online)

Keywords: *transport network, regrouping, road connections*

1. INTRODUCTION

The regrouping of a large number of people and equipment takes place several times a year in order to carry out training on the training ground important in the activities of the Polish Armed Forces (Kaszubowski, Mizer & Piasecki, 1970; Tarapata, 2000; Tarapata, 2001). The connections of the highlighted transport network are characterized by: the length, the speed at which they can be overcome, the capacities of connections, the intensity of traffic on connections and the costs of moving around connections (Hall, 1996; Suvin & Mallikarjuna, 2018; Basiewicz, Gołaszewski & Rudziński, 1998; Datka, Suchorzewski & Tracz, 1997). Paths included in the analyzed part of the transport network are divided into arcs (connections: two vertices are distinguished in the network as $\boldsymbol{a}$ and $\boldsymbol{b}$). The vertex $\boldsymbol{a}$ is the beginning of the regrouping and the vertex $\boldsymbol{b}$ is the final location of regrouping, pair ($\boldsymbol{a}, \boldsymbol{b}$) is the regrouping relationship. The characteristics of paths and arcs result directly from the characteristics of connections forming these paths.

Therefore, it is assumed (after Leszczyński, 1999; Jacyna, Kakietek & Przygocki, 2004) that the transport network is presented in the form of $\boldsymbol{G}$ graph, the form given as (1).

$$\boldsymbol{G} = \left\langle \boldsymbol{W}, \boldsymbol{M}, \boldsymbol{W}, \ \{c\}, \{d\}, \{x(ab)\} \right\rangle \tag{1}$$

wherein:
$\boldsymbol{W}$ – is a set of vertices (nodes) of the transport network. On the set of vertices, two vertices are distinguished, one $i = \boldsymbol{a}$ in which the regrouping starts and the second vertex $j = \boldsymbol{b}$ (different from the first highlighted and $\boldsymbol{a} \neq \boldsymbol{b}$) in which the regrouping takes place.
$\boldsymbol{M}$ – is a set of arcs numbers of the transport network (links between highlighted vertices).
$\boldsymbol{M}(i,j)$ – a set of connection numbers (arcs) between vertices i and j, given as formulae (2).

$$M(i,j) = \left\{ m:\ (i,m,j):\ (i,j) \in \boldsymbol{W} \times \boldsymbol{W},\ i \neq j,\ i \in \boldsymbol{W},\ j \in \boldsymbol{W},\ m \in \boldsymbol{M} \right\} \tag{2}$$

There may be many connections between the two vertices i and j (therefore many paths may exist while considering connections between the two locations).

An important challenge in optimization problems is to indicate which paths should be chosen in order to move optimally. Certainly, the condition (3) should be fulfilled.

$$\boldsymbol{M} = \bigcup_{(i,j) \in \boldsymbol{W} \times \boldsymbol{W}} M(i,j) \tag{3}$$

It is assumed that parameter c is mapped on the Cartesian product $\boldsymbol{W} \times \boldsymbol{M}$, which is given in formulae (4).

$$c : \boldsymbol{W} \times \boldsymbol{M} \rightarrow \Re^{+} \tag{4}$$

where the parameter $c((i,j),m) \in \Re^{+}$ is interpreted as cost of movement on path connection m between locations with numbers i and j. Later, notation $c(i,m,j)$ instead of $c((i,j),m)$ is used in order to maintain the predetermined interpretation.

It is also assumed that parameter d is mapped on the Cartesian product $\boldsymbol{W} \times \boldsymbol{M}$, which is given in formulae (5).

$$d : \boldsymbol{W} \times \boldsymbol{M} \rightarrow \Re^{+} \tag{5}$$

where the parameter $d((i,j),m) \in \Re^{+}$ is interpreted as cost of movement on road connection m between locations with numbers i and j. Later, notation $d(i,m,j)$ instead of $d((i,j),m)$ is used in order to maintain the predetermined interpretation.

There may exist many different paths and connections between the highlighted vertices $\boldsymbol{a}$ and $\boldsymbol{b}$ in the transport network. By $\boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})$ a set of road numbers between the highlighted vertices $\boldsymbol{a}$ and $\boldsymbol{b}$ is understood. The roads are numbered with the variable r. Therefore, a set of paths numbers is given as formulae (5).

$$\boldsymbol{P}(\boldsymbol{a},\boldsymbol{b}) = \left\{ r : r = \overline{1,P(\boldsymbol{a},\boldsymbol{b})} \right\} \tag{5}$$

where by $\boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})$ the number of different paths between the vertices $\boldsymbol{a}$ and $\boldsymbol{b}$ is determined (shortest paths are described e.g. in Tarapata, 2007). Through the path in the graph with the number r, $p((\boldsymbol{a},\boldsymbol{b}),r)$ is understood as the sequence of arcs in the form given as formulae (6).

$$\begin{gathered} p((\boldsymbol{a},\boldsymbol{b}),r) = \Big\langle \big((i(1),m(1)\ j(1)),r \big), \big((i(2),m(2),j(2)),r \big) \\ \big((i(s),m(s)\ ,j(s)),r \big),\ldots,\big(i(I(\boldsymbol{a})),m(S((\boldsymbol{a},\boldsymbol{b}),r)),j(J(b)),r \big) \Big\rangle \end{gathered} \tag{6}$$

The elements of formulae (6) meet the following conditions:

- $i(1) \equiv \boldsymbol{a}$ – vertex numbered as 1 is starting point of a designated path;
- $i(I(b)) \equiv \boldsymbol{b}$ – vertex numbered as $I(\boldsymbol{b})$ is ending point of a determined path;

- $\forall r \in \boldsymbol{P}(\boldsymbol{a},\boldsymbol{b}) \quad \forall\big(\big(i(s),m(s),j(s)\big),r\big) \in d\big((\boldsymbol{a},\boldsymbol{b}),r\big)$ $R\big(\big(i(s),m(s),j(s)\big),r\big) = 1; \quad s = \overline{1,P\big((\boldsymbol{a},\boldsymbol{b}),r\big)}$ – each ordered triple: vertex-arc-vertex is an element of a description for actual transport network, on which transport of goods is realised;
- $i(s+1) = j(s)$ – a starting point of the next triple, which defines the path, it should be the same vertex, which is the final one in the previous triple that defines the path; range of variability *s*, $s = \overline{1,\big(P\big((\boldsymbol{a},\boldsymbol{b}),r\big)-1\big)}$;
- $i(s) \neq i(s')$ – initial vertices in each of the triple which definines a path are different $s \neq s'$, $s,s' = \overline{1,P\big((\boldsymbol{a},\boldsymbol{b}),r\big)}$;
- $j(s) \neq j(s')$ – endpoints in each of the triple defining a path are different $s \neq s'$, $s,s' = \overline{1,P\big((\boldsymbol{a},\boldsymbol{b}),r\big)}$;
- $m(s) \neq m(s')$ – arcs (connections) in each triple defining a path are different $s \neq s'$, $s,s' = \overline{1,P\big((\boldsymbol{a},\boldsymbol{b}),r\big)}$.

For a given transport network there can be many regroup relations, i.e. many pairs $(\boldsymbol{a},\boldsymbol{b})$ may exist. By $\boldsymbol{R}$ the set of regroup relations is denoted – the set is given as formulae (7).

$$\boldsymbol{R} = \big\{\big(a(n),b(n)\big):\ n = \overline{1,N}\big\} \tag{6}$$

This means that *N* regrouping can be organized on the transport network. For each regrouping, rearrangements given in the paper can be repeated.

It is assumed that for the determined regrouping relationship $\big(a(n),b(n)\big)$, $n = \overline{1,N}$, the regrouping amount, i.e. the number of vehicles that is subjected to regrouping, is determined. This amount is denoted by $\alpha\big(a(n),b(n)\big)$.

It is assumed that on the Cartesian product $\boldsymbol{W} \times \boldsymbol{M}$ a mapping of *x* is given as formulae (7).

$$x : \boldsymbol{W} \times \boldsymbol{M} \to \Re^{+} \tag{7}$$

where $x\big((i,m,j),(\boldsymbol{a},\boldsymbol{b})\big) \in \Re^{+}$ has an interpretation of a number of vehicles traveling on an arc with *m* number connecting locations with numbers *i* and *j*. In the further part of the paper, notation $x(i,m,j)$ is used instead of $x((i,j),m)$, maintaining the predetermined interpretation.

The numerical values of $x((i,m,j),(\boldsymbol{a},\boldsymbol{b}))$ will be known only after solving the formulated optimization of the distribution of traffic flow.

2. ASSUMPTIONS ON DISPLACEMENT OF VEHICLES IN A GIVEN NETWORK IN A REGROUP RELATION

Non-negative assumptions to distribute a flow of vehicles to connections of the transport network is given as formulae (8).

$$\begin{gathered}\forall r \in \boldsymbol{P}(\boldsymbol{a},\boldsymbol{b}) \quad \forall m \in \boldsymbol{M} \quad \forall (i,j) \in \boldsymbol{W} \times \boldsymbol{W} \\ x\big((i(s),m(s),j(s)),r\big) R\big((i(s),m(s),j(s)),r\big) \geq 0 \\ s = \overline{1,P((\boldsymbol{a},\boldsymbol{b}),r)}\end{gathered} \tag{8}$$

Conditions for starting the regrouping task, for connecting paths with a fixed beginning in the vertex $i(s) = \boldsymbol{a}$ is given as formulae (9).

$$\sum_{r \in \boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})} \sum_{j(1) \in \Gamma a} \sum_{m(s) \in d(\boldsymbol{a},\boldsymbol{b},r)} x\big((i(s),m(s),j(s)),r\big) R\big((i(s),m(s),j(s)),r\big) \leq \alpha(\mathbf{a},\mathbf{b}) \tag{9}$$

Condition for not exceeding capacity of a connection *s*, $s = \overline{1,P((\boldsymbol{a},\boldsymbol{b}),r)}$ is given in formulae (10).

$$\sum_{(\boldsymbol{a},\boldsymbol{b}) \in \boldsymbol{E}} \sum_{r \in \boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})} \sum_{j(s) \in \Gamma i(s)} \sum_{m(s) \in d(\boldsymbol{a},\boldsymbol{b},r)} x\big((i(s),m(s),j(s)),r\big) R\big((i(s),m(s),j(s)),r\big) \leq d\big((i(s),m(s),j(s))\big) \tag{10}$$

Condition of vehicles flow adductivity is given as formulae (11).

$$\sum_{(\boldsymbol{a},\boldsymbol{b})\in\boldsymbol{E}}\ \sum_{r\in\boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})}\ \sum_{j(s)\in\Gamma i(s)}\ \sum_{m(s)\in d(\boldsymbol{a},\boldsymbol{b},r)} x\big((i(s),m(s),j(s)),r\big)R\big((i(s),m(s),j(s)),r\big)=x\big((i(s),m(s),j(s))\big) \tag{11}$$

Condition of vehicles flow retaining is given as formulae (12) for $x(i)\in\boldsymbol{V}$.

$$\begin{aligned}&\sum_{(\boldsymbol{a},\boldsymbol{b})\in\boldsymbol{E}}\ \sum_{r\in\boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})}\ \sum_{j(s)\in\Gamma^{-1}i(s)}\ \sum_{m(s)\in d(\boldsymbol{a},\boldsymbol{b},r)} x\big((i(s),m(s),j(s)),r\big)R\big((i(s),m(s),j(s)),r\big)-\\&+\sum_{(\boldsymbol{a},\boldsymbol{b})\in\boldsymbol{E}}\ \sum_{r\in\boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})}\ \sum_{j(s)\in\Gamma i(s)}\ \sum_{m(s)\in d(\boldsymbol{a},\boldsymbol{b},r)} x\big((i(s),m(s),j(s)),r\big)R\big((i(s),m(s),j(s)),r\big)=0\end{aligned} \tag{12}$$

It is assumed that set $\boldsymbol{V}$ is a set of vertices (nodes) of the transport network located inside this network. Set $\boldsymbol{A}$ is a collection of the start vertices of this network, while $\boldsymbol{B}$ is a set of final vertices of this network.

A vertex with the number $x(i)$ is a vertex inside the transport network $x(i)\in\boldsymbol{V}$, if the condition for it is met: $\Gamma\ x(i)\neq\varnothing\ \wedge\ \Gamma^{-1}\ x(i)\neq\varnothing$, i.e. to the vertex with the number $x(i)$ other arcs of this network arrive and also arcs from this vertex leaves to other vertices of the transport network.

The vertex with the number $x(i)$ is the starting point of the transport network, $x(i)\in\boldsymbol{A}$, if the condition for it is met: $\Gamma\ x(i)=\varnothing\ \wedge\ \Gamma^{-1}\ x(i)\neq\varnothing$, i.e. to the vertex with the number $x(i)$ none other arcs of this network arrive but arcs from this vertex leave to other vertices of the transport network.

The vertex with the number $x(i)$ is the end point of the transport network, $x(i)\in\boldsymbol{B}$, if the condition for it is met: $\Gamma\ x(i)=\varnothing\ \wedge\ \Gamma^{-1}\ x(i)\neq\varnothing$, i.e. to the vertex with the number $x(i)$ none other arcs of this network leave but arcs from this vertex arrive to other vertices of the transport network.

Condition of vehicles flow retaining is given as formulae (13) for $x(i)\in\boldsymbol{A}$.

$$\begin{aligned}&\sum_{(\boldsymbol{a},\boldsymbol{b})\in\boldsymbol{E}}\ \sum_{r\in\boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})}\ \sum_{j(s)\in\Gamma^{-1}i(s)}\ \sum_{m(s)\in d(\boldsymbol{a},\boldsymbol{b},r)} x\big((i(s),m(s),j(s)),r\big)R\big((i(s),m(s),j(s)),r\big)-\\&+\sum_{(\boldsymbol{a},\boldsymbol{b})\in\boldsymbol{E}}\ \sum_{r\in\boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})}\ \sum_{j(s)\in\Gamma i(s)}\ \sum_{m(s)\in d(\boldsymbol{a},\boldsymbol{b},r)} x\big((i(s),m(s),j(s)),r\big)R\big((i(s),m(s),j(s)),r\big)=\\&=-\alpha(\mathbf{a},\mathbf{b})\end{aligned} \tag{13}$$

Condition of vehicles flow retaining is given as formulae (14) for $x(i)\in\boldsymbol{B}$.

$$\sum_{(\boldsymbol{a},\boldsymbol{b})\in \boldsymbol{E}} \sum_{r\in \boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})} \sum_{j(s)\in \Gamma^{-1}i(s)} \sum_{m(s)\in d(\boldsymbol{a},\boldsymbol{b},r)} x\left(\left(i(s),m(s),j(s)\right),r\right) R\left(\left(i(s),m(s),j(s)\right),r\right) -$$
$$+ \sum_{(\boldsymbol{a},\boldsymbol{b})\in \boldsymbol{E}} \sum_{r\in \boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})} \sum_{j(s)\in \Gamma i(s)} \sum_{m(s)\in d(\boldsymbol{a},\boldsymbol{b},r)} x\left(\left(i(s),m(s),j(s)\right),r\right) R\left(\left(i(s),m(s),j(s)\right),r\right) = \tag{14}$$
$$= \alpha(\mathbf{a},\mathbf{b})$$

3. CRITERIA OF VEHICLES FLOWS IN REGROUP RELATIONS IN A GIVEN TRANSPORT NETWORK

Medium cost criterion, used to assess the distribution of traffic flow in a multimodal transport corridor, can be treated as an assessment of transport costs from the point of view of customers for transport services.

Most often, there are many decision makers in the transport system. The choice of a path by individual decision makers is made in such a way as to bring them the greatest possible benefits (minimize losses resulting from relocation).

In any relation $(\boldsymbol{a},\boldsymbol{b})$, where $(\boldsymbol{a},\boldsymbol{b}) \in \boldsymbol{E}$, there are many paths connecting the vertices ***a*** and ***b***. Choosing a specific path is the result of many decisions. This distribution of a traffic flow is an equilibrium, and is given as formulae (15).

$$c^{r,ab}(\boldsymbol{X}) = c^{r',ab}(\boldsymbol{X}) = min\, c^{r,ab}(\boldsymbol{X}) \tag{15}$$

where:

$c^{r,ab}(\boldsymbol{X})$ – is interpreted as the value of average costs for vehicles flow transfer on a *r*-th path in relation $(\boldsymbol{a},\boldsymbol{b})$;

$c^{r',ab}(\boldsymbol{X})$ – is interpreted as the value of average costs for the vehicles flow transfer on a *r'*-th path in relation $(\boldsymbol{a},\boldsymbol{b})$;

X – a vector with elements x_{ij}, which are interpreted as the size of the vehicles flow moved on $m(s)$ contained between the vertices $i(s)$ and $j(s)$ on transport network.

The above expression reaches the minimum value for each transport relation $(\boldsymbol{a},\boldsymbol{b}) \in \boldsymbol{E}$ and for different paths *r* and *r'* of this relation, where $r,r' \in \boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})$. It also means that the average cost of travel on each of paths used in this relation is the same. Therefore, in accordance to the conditions of equilibrium, the minimum value of the average cost is also an average cost of moving vehicles flow within each paths of transport relation. It should also be noted that this value is characteristic for a given transport network layout and for a given transport demand size [$x^{\boldsymbol{ab}}$]. Optimization of the distribution of traffic flow according to the medium cost criterion allows to predict the behavior of customers of transport services in situations that leave freedom to choose a variant of demands generated by them.

4. OPTIMIZING TASKS FOR VEHICLES FLOW

4.1. Optimization task of vehicles flow with average cost criterion

The function criterion should be minimized as it is given in (16), if the following restrictions are met.

$$\mathrm{F}\left(\boldsymbol{X}\right) = c^{r,ab}\left(\boldsymbol{X}\right) = c^{r',ab}\left(\boldsymbol{X}\right) = min\, c^{r,ab}\left(\boldsymbol{X}\right) \tag{16}$$

Non-negative assumptions to distribute a flow of vehicles to connections of the transport network is given as formulae (17).

$$\begin{gathered}\forall r \in \boldsymbol{P}(\boldsymbol{a},\boldsymbol{b}) \quad \forall m \in \boldsymbol{M} \quad \forall\left(i,j\right) \in \boldsymbol{W} \times \boldsymbol{W} \\ x\Big(\big(i(s),m(s),j(s)\big),r\Big)R\Big(\big(i(s),m(s),j(s)\big),r\Big) \geq 0\end{gathered} \tag{17}$$

Conditions for starting a regrouping task, for connecting paths with a fixed beginning in the vertex and $i\left(s\right) = \boldsymbol{a}$ is given as formulae (18).

$$\sum_{r\in\boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})}\ \sum_{j(1)\in\Gamma a}\ \sum_{m(s)\in d(\boldsymbol{a},\boldsymbol{b},r)} x\Big(\big(i(s),m(s),j(s)\big),r\Big)R\Big(\big(i(s),m(s),j(s)\big),r\Big) \leq \alpha(\mathbf{a},\mathbf{b}) \tag{18}$$

Condition for not exceeding capacity of a connection *s*, $s = \overline{1,P\big((\boldsymbol{a},\boldsymbol{b}),r\big)}$ is given as formulae (19).

$$\sum_{(\boldsymbol{a},\boldsymbol{b})\in\boldsymbol{E}}\ \sum_{r\in\boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})}\ \sum_{j(s)\in\Gamma i(s)}\ \sum_{m(s)\in d(\boldsymbol{a},\boldsymbol{b},r)} x\Big(\big(i(s),m(s),j(s)\big),r\Big)R\Big(\big(i(s),m(s),j(s)\big),r\Big) \leq d\Big(\big(i(s),m(s),j(s)\big)\Big) \tag{19}$$

Condition of vehicles flow adductivity is given as formulae (20).

$$\sum_{(\boldsymbol{a},\boldsymbol{b})\in \boldsymbol{E}} \sum_{r\in \boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})} \sum_{j(s)\in \Gamma i(s)} \sum_{m(s)\in d(\boldsymbol{a},\boldsymbol{b},r)} x\Big(\big(i(s),m(s),j(s)\big),r\Big) R\Big(\big(i(s),m(s),j(s)\big),r\Big) = x\Big(\big(i(s),m(s),j(s)\big)\Big) \tag{20}$$

Condition of vehicles flow retaining is given as formulae (21) for $x(i)\in \boldsymbol{V}$.

$$\begin{aligned}&\sum_{(\boldsymbol{a},\boldsymbol{b})\in \boldsymbol{E}} \sum_{r\in \boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})} \sum_{j(s)\in \Gamma^{-1} i(s)} \sum_{m(s)\in d(\boldsymbol{a},\boldsymbol{b},r)} x\Big(\big(i(s),m(s),j(s)\big),r\Big) R\Big(\big(i(s),m(s),j(s)\big),r\Big) - \\ &+ \sum_{(\boldsymbol{a},\boldsymbol{b})\in \boldsymbol{E}} \sum_{r\in \boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})} \sum_{j(s)\in \Gamma i(s)} \sum_{m(s)\in d(\boldsymbol{a},\boldsymbol{b},r)} x\Big(\big(i(s),m(s),j(s)\big),r\Big) R\Big(\big(i(s),m(s),j(s)\big),r\Big) = 0\end{aligned} \tag{21}$$

Condition of vehicles flow retaining is given as formulae (22) for $x(i)\in \boldsymbol{A}$.

$$\begin{aligned}&\sum_{(\boldsymbol{a},\boldsymbol{b})\in \boldsymbol{E}} \sum_{r\in \boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})} \sum_{j(s)\in \Gamma^{-1} i(s)} \sum_{m(s)\in d(\boldsymbol{a},\boldsymbol{b},r)} x\Big(\big(i(s),m(s),j(s)\big),r\Big) R\Big(\big(i(s),m(s),j(s)\big),r\Big) - \\ &+ \sum_{(\boldsymbol{a},\boldsymbol{b})\in \boldsymbol{E}} \sum_{r\in \boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})} \sum_{j(s)\in \Gamma i(s)} \sum_{m(s)\in d(\boldsymbol{a},\boldsymbol{b},r)} x\Big(\big(i(s),m(s),j(s)\big),r\Big) R\Big(\big(i(s),m(s),j(s)\big),r\Big) = \\ &= -\alpha(\mathbf{a},\mathbf{b})\end{aligned} \tag{22}$$

Condition of vehicles flow retaining is given as formulae (23) for $x(i)\in \boldsymbol{B}$.

$$\begin{aligned}&\sum_{(\boldsymbol{a},\boldsymbol{b})\in \boldsymbol{E}} \sum_{r\in \boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})} \sum_{j(s)\in \Gamma^{-1} i(s)} \sum_{m(s)\in d(\boldsymbol{a},\boldsymbol{b},r)} x\Big(\big(i(s),m(s),j(s)\big),r\Big) R\Big(\big(i(s),m(s),j(s)\big),r\Big) - \\ &+ \sum_{(\boldsymbol{a},\boldsymbol{b})\in \boldsymbol{E}} \sum_{r\in \boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})} \sum_{j(s)\in \Gamma i(s)} \sum_{m(s)\in d(\boldsymbol{a},\boldsymbol{b},r)} x\Big(\big(i(s),m(s),j(s)\big),r\Big) R\Big(\big(i(s),m(s),j(s)\big),r\Big) = \\ &= \alpha(\mathbf{a},\mathbf{b})\end{aligned} \tag{23}$$

The result of the solution designation $\boldsymbol{X}^* = \Big[x^*\Big(\big(i(s),m(s),j(s)\big),r\Big)\Big]$ that meets the assumptions and conditions imposed on a vehicle flow is acceptable solution.

The optimal solution for a task is distribution of traffic flow, taking into account quality criterion of a solution.

4.2. Optimization task of vehicles flow with cost criterion

The function criterion given as formulae (24) should be minimized, if the restrictions (17)–(23) are met.

$$\sum_{r\in \boldsymbol{P}(\boldsymbol{a},\boldsymbol{b})} \sum_{s\in \boldsymbol{S}(\boldsymbol{a},\boldsymbol{b})} \sum_{j(s)\in \Gamma a} \sum_{m(s)\in d(\boldsymbol{a},\boldsymbol{b},r)} x\Big(\big(i(s),m(s),j(s)\big),r\Big)\times$$
$$\times x\Big(\big(i(s),m(s),j(s)\big),r\Big)R\Big(\big(i(s),m(s),j(s)\big),r\Big)\rightarrow \min \qquad (24)$$

5. CONCLUSION

The paper deals with the problem of the distribution of traffic flow of transport in a regrouping region. The loads subjected to regrouping are divided. Potential means of transport that can be used for regrouping are identified and also the technical and economic parameters of the transport network of a regrouping region, in particular in the transport corridors, are determined. Optimization tasks are planned for the distribution of means of transport in the region of regrouping in the sense of minimizing average costs, in the sense of minimizing external costs and regrouping time (the last one is a matter of future research). The application of the method for determining the traffic flow confirms the validity of the made assumptions. It is expected that, as a result of the optimization tasks appropriate to the decision-making situations, the rearrangements would be determined by those transport corridors (paths) in the case of which cost of regrouping would be minimal. Therefore, it will be possible to reduce cost of regrouping for each form of regrouping. The software for the method of determining the rearrangement corridors (numerical model) would allow evident financial effects for each region of regrouping. Numerical model will be the subject matter of future research, as well as verification of the analytical model presented in this paper and validation of future numerical model.

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BIOGRAPHICAL NOTES

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